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Three-Dimensional Validation of an Integrated Estimation/Guidance Algorithm Against Randomly Maneuvering Targets

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I. Introduction

SUCCESSFUL interception of maneuvering tactical ballistic missiles (TBM), requiring very small miss distances or even a direct hit, has been a challenging problem since their reintroduction as a terror weapon in the 1991 Gulf War. At the Faculty of Aerospace Engineering of the Technion—Israel Institute of Technology, a multiyear research has been conducted investigating the difficulties of effective ballistic missile defense (BMD). It has been aimed to identify and correct the deficiencies of the conservative common practice in the estimation/guidance law design. In the last years, several papers dealing with this yet unsolved problem were published [1–9]. In a recently published work [9], an integrated estimation/guidance design paradigm against randomly maneuvering targets was introduced, based on several innovative concepts that provide robust satisfactory homing performance. The main ideas involved in this revolutionary design concept were developed and tested by using a simplified linearized planar (horizontal) constant-speed model of the interception scenario. A set of Monte Carlo simulations demonstrated that in such scenarios this new approach yields a substantial improvement in homing accuracy, compared with earlier results.

The simplified mathematical model of the interception scenario raised two critical questions. The main concern has been whether an algorithm that was developed for a time-invariant planar model will perform well in a realistic endoatmospheric BMD scenario, which is inherently three-dimensional with variable speeds and maneuverability constraints. The other question related to target maneuver types. Because the study in [9] was focused on a single maneuver structure (a bang–bang type with random switching time), can the algorithm deal with other types of feasible target maneuvers?

The objective of the present Note is to provide answers to these questions by reporting the results of an extensive simulation study evaluating the validity of the integrated estimation/guidance algorithm of [9] in a set of three-dimensional nonlinear endoatmospheric BMD interception scenarios. The test was performed against two types of the most stressing random target maneuvers: namely, the bang–bang-type with random switching time (used in [9]) and a periodical random-phase maneuver discussed in some recent work [10]. Although an eventual threat can generate other maneuver structures, they will be less effective and therefore less likely.

Early distinction between the two types of evasive maneuvering is one of the new elements of this study, modifying the originally planar algorithm of [9] to suit a realistic three-dimensional environment. The limited length of a Technical Note did not allow elaborating on the theory and the underlying mathematics. More details can be found in the references and, in particular, in [9].

The structure of this Note is the following. In the next section, the three-dimensional BMD problem is formulated. In Sec. III, the main elements of the integrated estimation/guidance algorithm used in [9] are summarized and the modifications needed for its application in a realistic three-dimensional BMD are outlined. This is followed by the scenario data, including the generic target and interceptor models used in this validation study. The three-dimensional simulation philosophy and the results of extensive Monte Carlo simulations are presented in Sec. V. Summary and conclusions are offered in Sec. VI.

II. Problem Statement

A. Scenario Description

A realistic three-dimensional endoatmospheric BMD scenario with time-varying speeds and acceleration limits against randomly maneuvering TBMs is considered. The target is a generic TBM with aerodynamic control, capable of performing evasive maneuvers. It is assumed to be launched from the distance of 600 km on a minimum-energy trajectory. The interceptor is a generic roll-stabilized two-stage solid rocket missile that has two identical guidance channels for aerodynamic control (skid to turn). It is assumed to be launched in a point defense task, based on a firing solution for a given interception altitude. The relative initial endgame geometry is near to a head-on engagement. The homing endgame starts shortly before interception, as soon as the onboard seeker of the interceptor succeeds to acquire the target. It is assumed that, at this moment, the initial heading error with respect to a collision course is small and neither the interceptor nor the target is maneuvering. These assumptions allow the linearization of the interception geometry and, consequently, [11] the decoupling the three-dimensional equations of motion in two identical sets in perpendicular planes.

B. Information Structure

It is assumed that the interceptor measures range and range rate with good accuracy, allowing the computation of the time-to-go. However, the measurements of the line-of-sight angle are corrupted by a zero-mean white Gaussian angular noise. The interceptor's own acceleration is accurately measured, but the target acceleration has to be estimated based on the available measurements. The target has no information on the interceptor but, being aware of an interception attempt, can start evasive maneuvers at any time during the endgame and change the direction of the maneuver randomly.

C. Lethality Model

The objective of the interception is the destruction of the attacking TBM. In this investigation, the probability of destroying the target is determined by the following simplified lethality function:

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$$P_d = \begin{cases} 1, & M \le R_k \\ 0, & M > R_k \end{cases} \qquad (1)$$

where R_k is the lethal (kill) radius of the warhead and M is the miss distance. This model assumes an overall reliability of 100% of the entire guidance system.

D. Performance Index

The natural (deterministic) cost function of the interception engagement is the miss distance. Because of the noisy measurements and the random target maneuvers, the miss distance is a random variable with an a priori unknown probability distribution function. Based on the lethality function (1), the efficiency of a guided missile strongly depends on the lethal radius R_k of its warhead. One of the possible figures of merit is the single-shot kill probability (SSKP) for a given warhead, defined by

$$\mathrm{SSKP} = E\{P_d(R_k)\} \qquad (2)$$

where E is the mathematical expectation taken over the entire set of noise samples against any given feasible target maneuver. The objective of the guidance is to maximize this value. An alternative figure of merit is the smallest possible lethal radius R_k that guarantees a predetermined probability of success. In several recent studies [2–9], the required probability of success has been assumed as 0.95, yielding the following performance index:

$$J = R_k = \arg\{\mathrm{SSKP} = 0.95\} \qquad (3)$$

Practically, the homing performance is evaluated by a large set of Monte Carlo simulations, leading to obtain the cumulative probability distribution function of the miss distance that also presents the relationship between the preceding two defined crisp figures of merit. This approach allows a comprehensive homing performance comparison of different cases.

E. Problem Formulation

Interception problems in the past were formulated mostly as optimal control problems, assuming the knowledge of the current and future target maneuver time history. There is a basic deficiency in such a formulation, because target maneuvers are independently controlled and future target maneuver time history (or strategy) cannot be predicted. The scenario of intercepting a maneuverable target has to be formulated as a zero-sum differential game of pursuit–evasion [12]. In this formulation, there are two independent controllers, and the cost function is minimized by one of them and maximized by the other. So far, several deterministic zero-sum pursuit–evasion game models [13–15] were solved. These game solutions simultaneously provided the interceptor's guidance law (the optimal pursuer strategy), the worst target maneuver (the optimal evader strategy), and the resulting guaranteed miss distance (the saddle-point value of the game). The optimal guidance law used in this study is based on a perfect-information linear time-varying game solution with bounded controls [3], requiring the knowledge of the current target maneuver.

III. Estimation/Guidance Algorithm

A. Planar Algorithm

In [9] a planar constant-speed scenario was investigated, considering a single type of evasive maneuver: namely, a randomly switched bang–bang-type evasion with maximum admissible amplitude. An integrated estimation/guidance algorithm was structured to deal with such interceptions. It had several innovative elements, both in the estimation and the guidance phases, as well as in their integration.

1. Estimation

In a guided missile system the estimator performs dual tasks: the task of a filter (smoothing and averaging the noise-corrupted measurements) and the task of an observer (reconstructing the nonmeasured or nonmeasurable state variables, such as target acceleration). Satisfactory performance of each task requires a different type of estimator design. Because no single estimator could satisfy the contradictory requirements of homing accuracy (a narrow bandwidth for good filtering and a large bandwidth for following the eventual changes), in the new algorithm, the different tasks performed by a classical estimator were separated and assigned to different elements within a corporate estimation system.

The main task directly affecting the homing accuracy is the estimation of the state variables (including the target acceleration) involved in the guidance law. This task was performed by an estimator of narrow bandwidth [namely, a Kalman filter with a shaping filter of narrow bandwidth, based on an exponentially correlated acceleration (ECA) model [16]], assuming that the direction of the target maneuver is identified. Thus, at the beginning of the endgame, such identification was performed. The filters used for this task were of relatively large bandwidth, to complete the identification as fast as possible. It was demonstrated that such an estimator combined with an appropriate guidance law can yield very small miss distances.

If the target changes the maneuver direction during the endgame at an early phase (sufficiently far away from the end), the slow estimator can identify it, converge to the correct values, and provide accurate information to the guidance law. However, if the change takes place after a critical time-to-go, there is not enough time available for this process and large miss distances are created. This problem can be alleviated by using a separate fast detector device and a set of tuned estimators.

In an earlier paper also dealing with a planar scenario [17], a multiple-model estimator is described, in which each ECA model assumed a different timing of the direction reversal (switch) of a bang–bang-type maneuver. Using such an estimator tuned to the correct switch eliminates the delay and yields excellent homing performance. Even if the switch occurs shortly after the time anticipated by the estimator, good performance is obtained. Because of this robustness property, a few adequately tuned estimators can cover the range of interest. These state estimators did not have a too-narrow bandwidth to allow convergence in a short time.

2. Guidance

The guidance law used for a planar constant-speed scenario was derived based on the following assumptions:

1) The engagement between the interceptor (pursuer) and the maneuvering target (evader) takes place in a horizontal plane.

2) Both the interceptor and the maneuvering target have constant speeds V_j and bounded lateral accelerations $|a_j| < (a_j)^{\max}$ $(j = E, P)$.

3) The maneuvering dynamics of both vehicles can be approximated by first-order transfer functions with time constants τ_P and τ_E, respectively.

4) The relative interception trajectory can be linearized with respect to the initial line of sight.

The set of assumptions 1–4 allowed casting the problem to the canonical form of linear games, from which a reduced-order game with only a single state variable, the zero-effort miss distance, denoted by Z, was obtained. As the independent variable of the problem, the time-to-go t_{go} was selected. The solution of this game is determined by two parameters of physical significance: the interceptor/target maximum maneuverability ratio $\mu = (a_P)^{\max}/(a_E)^{\max}$ and the ratio of the target/interceptor time constants $\varepsilon = \tau_E/\tau_P$. The cost function of the game is the miss distance and the bounded lateral accelerations are respected by limiting the acceleration commands accordingly. In such a formulation, the guidance law [15], denoted as DGL/1, is of the bang–bang type based on the sign of the zero-effort miss distance.

3. Guidance Law Modifications

During the development of the algorithm in [9] it was found that if the eventual change of direction occurs near the end, then (due to the inherent detection delay and the remaining short time) the interceptor is unable to reach its maximum lateral acceleration and cannot correct the guidance error generated during the delay. This deficiency was alleviated by increasing the lateral acceleration command for small values of time-to-go in a way that the actual acceleration limits are respected. Further homing improvement was achieved by introducing a time-varying dead-zone version of the signum function in the guidance law for the period when the tuned estimators were used. This modification, reducing the error created during the detection delay, was used only until the maneuver direction change was detected.

B. Three-Dimensional Algorithm

In the present Note, dealing with three-dimensional endoatmospheric BMD scenarios, the scope of the study is extended to include two basic types of target maneuver models. The first one is a slowly varying, piecewise-continuous, planar bang–bang maneuver, assuming a roll-stabilized target. For the sake of comparability with [9], the maneuvers are oriented in a horizontal plane. The amplitude of the maneuver is monotonically increasing as the target descends to lower altitudes. The second type of maneuver assumes a rolling target with a fixed angle of attack in body coordinates, creating a barrel-roll (spiral) type of maneuver of time-varying (monotonically increasing) amplitude. A spinning aerodynamically stable reentry vehicle will inherently perform a similar maneuver. (Although, in this case, the maneuver frequency is time-varying, in the endgame of short duration, it can be considered as constant.)

Each type of maneuver requires a different type of estimator. The distinction between the two different types of target maneuvers considered in this study is an essential new element in this study.

1. Estimation

The group of estimators for the bang–bang-type maneuver is similar to the one used in [9]. The only difference is that instead of a constant acceleration command, a monotonically increasing command is anticipated. The second type of evasive target maneuver creating a barrel-roll (spiral) type of trajectory of time-varying radius requires a different estimator. Because this is a typical three-dimensional maneuver, two planar estimators have to be used to estimate the projections of the motion in two perpendicular planes. Within each plane, the motion is periodical with random phase, and therefore the appropriate shaping filter has to be of the second order [18], assuming a known maneuver frequency. Such a Kalman filter estimates not only the target acceleration, but also its time derivative (the jerk). If the maneuver frequency is correctly predicted, the output of such an estimator converges well to the actual maneuver, even if its amplitude is slowly varying. As a consequence, the homing accuracy is satisfactory. However, if the frequency used in the estimator is incorrect, the estimation is degraded and the homing performance is poor. Because the number of estimators used in the bank is finite, the bandwidth of each periodical estimator has to be tuned to allow a reasonable error in the predicted frequency without a great compromise in the homing accuracy.

The first phase of the estimation, to be carried out as soon as the interception endgame starts, is devoted to identifying the proper target maneuver model by using a multiple-model structure [19]. The filters for this task are of large bandwidth, to complete the model identification as fast as possible. The clear distinction between the two different maneuver types is of major importance.

Once the decision between the two types of maneuver is made, the next step of the target model identification for each one becomes different. For the bang–bang-type target maneuver, the direction of the maneuver has to be found. For a spiral maneuver, the frequency range of the model has to be identified with a reasonable accuracy. In both cases, the initial amplitude of the maneuver has to be also estimated, based on an intelligent guess.

Identifying the approximate value of the frequency for a periodical maneuver requires a rather large number of estimator models spanning the range of the expected frequency domain. The separation of the models has to be such that a reasonable frequency inaccuracy does not create substantial performance reduction.

After the completion of the model identification, the appropriate state estimator of narrow bandwidth is selected to forward information to the guidance law. Continuous computation of the a posteriori probabilities are used to confirm the correctness of the selection. For a spiral maneuver, no dramatic changes in the model are expected. For the bang–bang-type target maneuver, the eventual change of direction is expected to be detected by a sufficiently fast detector, leading to the use of the nearest tuned estimator as in [9]. Because the development of such a fast detector is yet incomplete, the detection delay is parameterized in the present Note.

2. Guidance Law

Against a target performing a pure periodical maneuver, the optimal guidance law was presented in [10]. However, this guidance law is not robust enough for the realistic case of time-varying frequencies and maneuver amplitudes. Guidance laws based on differential game formulations are much more robust with respect to these uncertainties.

In the first phase of the endgame, until the target maneuver is identified, a simple estimator of narrow bandwidth and a differential game based guidance law, denoted as DGL/0 [11], not requiring knowledge of the target maneuver, are used.

The guidance law DGL/1 used in [9], in which a planar scenario was investigated, was derived based on the assumption of constant speeds. Because in a three-dimensional scenario the velocities are not constant and the bounds of the lateral accelerations vary with speed and altitude, the model of the game has to be modified. In a perfect-information game, assumption 2 of Sec. III.A.2 is replaced by assuming that profiles of the time-varying parameters are known along a nominal trajectory. Such a model is suitable for the analysis of a realistic BMD scenario. The solution of this game [3] is qualitatively similar to [15], but its outcome depends strongly on the respective velocity/maneuverability profiles of the players and, obviously, the value of μ is not constant. Because of the time-varying profiles, the expressions of the zero-effort miss distance and the other elements of the solution become more complex. These expressions were developed in [3]. In spite of this (algebraic) complexity, the implementation of the optimal missile guidance law, denoted as DGL/E, does not present essential difficulties. It requires, of course (in addition of the perfect knowledge of the current lateral acceleration of the target), the velocity and maneuverability profiles in the endgame that can be precalculated along a nominal trajectory.

The guidance law modifications of Sec. III.A.3 are preserved in this study and were applied using DGL/E if the type of target maneuver was not periodical.

IV. Scenario Data

Endoatmospheric interception scenarios terminating between altitudes of 20–30 km with an initial range of 20 km are considered.

As mentioned earlier, the target is a generic TBM with aerodynamic control, performing either spiral or horizontal bang–bang evasive maneuvers. It is assumed to be launched from the distance of 600 km on a minimum-energy trajectory. It is characterized by a ballistic coefficient $\beta = 5000\ \mathrm{kg/m^2}$ and a trimmed lift-to-drag ratio $\Lambda = 2.6$. Its velocity at reentry of an altitude of 150 km is $V_{e0} = 1720\ \mathrm{m/s}$ with a flight-path angle of $\gamma_{e0} = -18$ deg and a horizontal distance of 210 km from its surface target.

The interceptor is a generic roll-stabilized two-stage solid rocket missile that has two identical guidance channels for aerodynamic control (skid to turn). Its seeker provides angular measurements at a sampling rate of 100 Hz. These angular measurements are corrupted by zero-mean white Gaussian angular noise of constant amplitude with a standard deviation of 0.1 mrad.

Both stages of the rocket motor have a specific impulse of $I_{sp} = 250$ s. The propulsion, mass, and aerodynamic data of the two stages are summarized in Table 1.

Table 1 Interceptor data

	t_b, s	T, kN	m_0, kg	SC_D, m^2	$SC_{L\max}$, m^2
First stage	6.5	229	1540	0.10	0.24
Second stage	13	103	781	0.05	0.20

The second stage of the rocket motor is ignited with a delay to guarantee that, for any interception altitude, the endgame terminates with a positive longitudinal acceleration and nondecreasing maneuverability. The velocity profiles for different interception altitudes are shown in Fig. 1.

During the endgame, the maneuverability of the target is monotonically increasing, and as a consequence, the value of the interceptor/target maneuverability ratio, denoted by μ, is monotonically decreasing, as shown in Fig. 2. The maneuvering dynamics of the interceptor (pursuer) and the target (evader) are approximated by first-order transfer functions with equal time constants $\tau_P = \tau_E = 0.2$ s.

V. Simulation Results

As shown in Fig. 2, the duration of the endgame is slightly more than 4 s. The distinction between periodical and planar maneuvers has been successfully completed for all cases during the first second of the engagement. Identification of the actual maneuver frequency was carried out by a bank of multiple-model estimators using 12 estimator models spanning the range of 0–2 Hz. Successful completion of this phase required a longer time, but ended at least 1 s (5 time constants) before endgame termination. Assuming Gaussian noise for an interception altitude of 25 km, the test of the combined estimation/guidance scheme against a very large set of randomly selected target maneuvers yielded encouraging results. For all maneuver frequencies between 0.05–2.0 Hz, the average miss distances were less than 20 cm. Homing accuracy statistics expressed by the cumulative miss distance distribution, shown in Fig. 3, indicates that 95% of the miss distances are less than 32 cm.

In the sequel, the cumulative distribution of some particular cases are also presented: 1) homing accuracy against random horizontal bang–bang maneuvers (using the integrated estimation/guidance scheme of [9]) at different interception altitudes and 2) homing accuracy against spiral target maneuvers of different roll rates p_0, using matched and unmatched periodical estimators at the interception altitude of 25 km.

In Fig. 4, the results against random horizontal bang–bang maneuvers with uniformly distributed reversal time (switch) with 1000 Monte Carlo simulation at three interception altitudes (15, 20, and 25 km), assuming a detection delay of 0.1 s, are shown. The miss distances of 20–30 cm (or less) for 95% of the cases are similar to the results of [9], indicating the potential for satisfying a hit-to-kill requirement.

The results against random-phase spiral target maneuvers with different roll rates are presented in Fig. 5. It can be seen that if the maneuver frequency of the roll rate and the frequency used in the estimator are perfectly matched, the homing accuracy is independent of the roll rate (at least in the tested region) and similar to the accuracy against the bang–bang maneuvers. However, if the frequency used in

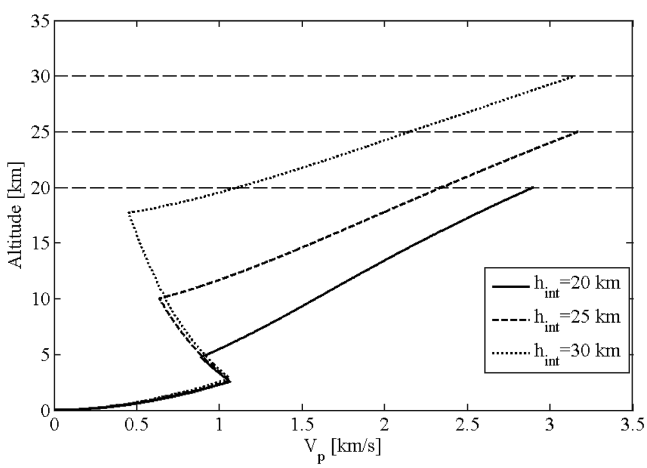

Fig. 1 Interceptor velocity profiles.

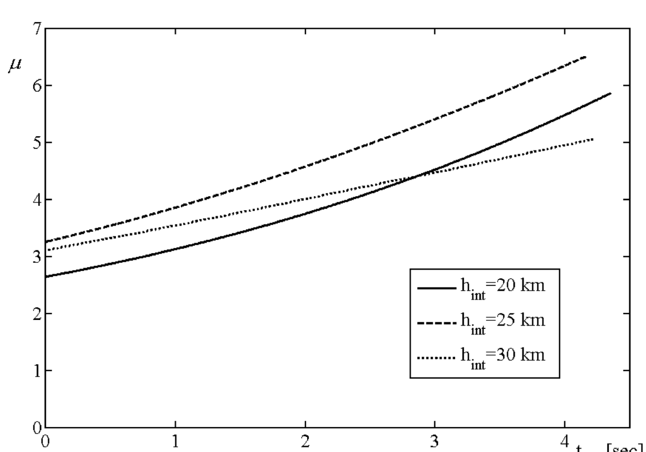

Fig. 2 Endgame maneuverability ratio.

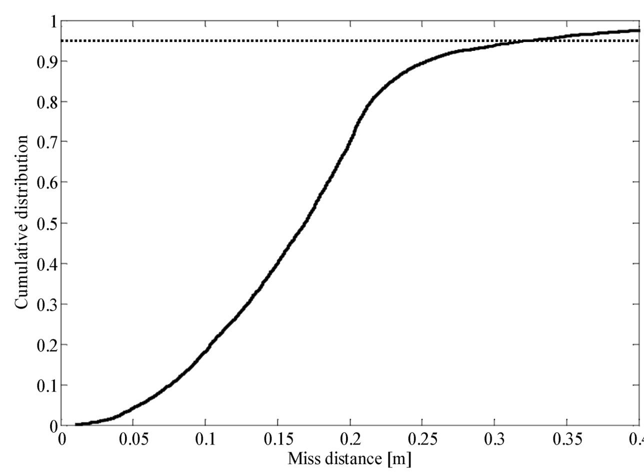

Fig. 3 Homing accuracy against randomly selected target maneuvers.

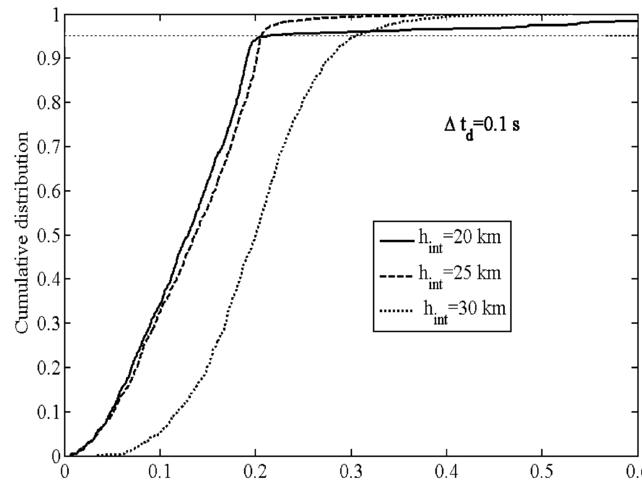

Fig. 4 Homing accuracy against random horizontal bang–bang maneuvers.

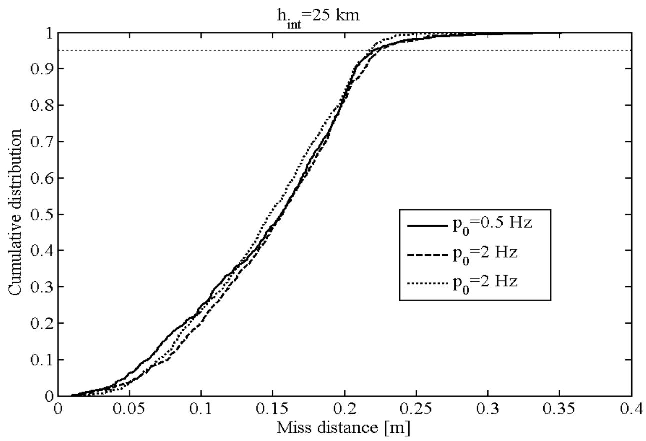

Fig. 5 Homing accuracy against random-phase spiral maneuvers (matched estimators).

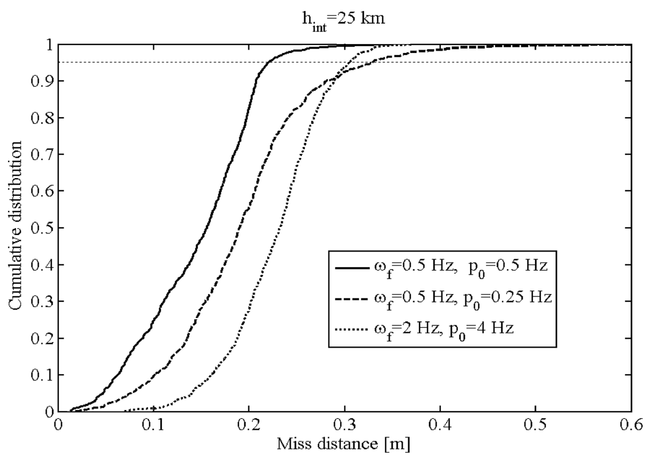

Fig. 6 Homing accuracy against random spiral maneuvers (unmatched estimators).

the estimator is different from the actual roll rate, the homing accuracy is degraded, as shown in Fig. 6.

VI. Conclusions

The simulation results shown by Figs. 3–6 justify the additional development effort needed to confirm the validity of the integrated estimation/guidance approach, based on a simplified planar constant-speed model in a generic (but realistic) theater ballistic missile defense scenario against randomly maneuvering targets. By using comparable generic target and interceptor models, it is shown that in realistic three-dimensional interception scenarios, a homing accuracy similar to that found in planar engagements [9] can be achieved. A successful interception requires, however, an early identification of the target maneuver type. The results also emphasize the severe performance degradation caused by using an incorrect (not adapted) estimator in the homing process. Moreover, against spiral maneuvers, correct estimation of the maneuvering frequency is needed.

The examples tested in the validation study clearly demonstrate that an integrated estimation/guidance approach, as it was developed, can not only lead to a substantial homing accuracy improvement compared with earlier results, but also has the potential to satisfy the hit-to-kill requirement against two types of stressing evasive target maneuvers.

The applicability of the new design approach using the integrated estimation/guidance algorithm is twofold: 1) upgrading the homing performance of existing interceptors without requiring hardware modifications and 2) designing new cost-effective interceptors with reduced hardware requirements.

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